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RESEARCH ARTICLE

FREQUENCY ANALYSIS OF ANNUAL AND MAXIMUM DAILY RAINFALL IN HA'IL AREA USING PROBABILITY DISTRIBUTION MODELS

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ABSTRACT

This study uses the Frequency analysis of the maximum daily and annual rainfall recorded during the period 1976-2020 at eight rain-measuring stations in the Ha'il region: Jubbah (106H-196), Baqa'a (103H-193), Simirah (105H-793), Uqlah Ibn Jibrin (208H-508) and Faydat Ibn Suwaylim (108H-198), Al Ha'it (111H-797), Al Ghazalah (215H-812) and Ha'il (101H-191), currently supervised by the Ministry of Environment, Water and Agriculture (MEWA). These stations were selected due to their locations in the Hail region, which is in great urban development causing an increase in demand for water resources to meet the growing population in urban centers. This paper compares four probability distributions (Normal, Log Normal, Gumbel and Exponential) using the rainfall data recorded in eight rain stations located in Ha'il area. Sets of data cover a period of about 45 years. Therefore, this study aims to estimate the maximum daily and annual rainfall by applying 4-four probability distributions, which are the Normal, the Log-Normal, the Exponential, and the Extreme Values-type 1, using the (Chow) model estimation. Results showed that Gumbel (EV 1) distribution has the least critical values in the tests. Hence, it is considered the best fit distribution for annual rainfall in Simirah and Baq'a. Furthermore, based on the results, it is evident that the exponential distribution is the best in Al Ha'it, Faydat Ibn Suwaylim, Al Ghazalah, Ha'il, and Jubbah. Whilst, the log normal distribution is the best in Al Uqlat. The results also showed that Gumbel (EV 1) distribution has the least critical values in the tests. Thus, it is regarded as the best-fit distribution for the maximum daily rainfall in Simirah and Ha'il, and the normal distribution is considered the best for the rest of the rain stations.

KEYWORDS

Ministry of Environment, Water and Agriculture (MEWA), Gumbel (EV 1), rainfall

1. INTRODUCTION

In arid regions environments, knowledge of the spatial and temporal variability and trends of rainfall are important for estimating water resources and planning under the global warming effects. Several studies were carried out on the analysis of rainfall variability and distribution in time as well as in space Saudi Arabia. Furthermore, the rain variability causes an impact on human life in urban cities and cultural activities in rural agglomerations. Accordingly, analyzing the temporal and spatial wet and dry spell durations as well as rainfall intensity on the daily basis necessary for effective future urban planning along with water resources management.

Since rainfall is erratic and varies with time and space in arid zones; such as Ha'il area, yet it is possible to predict design rainfall accurately for certain return periods using various probability distributions (Upadhaya and Singh, 1998). Consequently, the rainfall frequency analysis is the main input for water resources management in arid regions. It's required for various engineering designs such as hydraulic structures, water conservation structures, bridges and culverts, canals, stormwater sewers and road drainage systems in urban cities. The detailed statistical analysis of each rain station is essential to estimate the value of rain amounts, which can be used in engineering structures, and for agricultural planning. In this context, the present study uses the frequency analysis of daily maximum rainfall data and yearly rainfall of eight rain stations extended over the Ha'il area. The rainfall data for 45 years is collected to evaluate return period values of rainfall. Seven different probability distribution models (Gamble's extreme value type I, Log Pearson type III, Lognormal, Normal, Exponential, Pearson type III and Gamma distribution) were used to assess maximum daily rainfall and yearly rainfall. Chi-squared tests were used for both the goodness and validity of fit of the probability distributions.

The frequency analysis can be used for predicting the occurrence of future events of rainfall from the available data by applying the statistical methods (Kumar and Kumar, 1989). Several studies of rainfall distributions in space and time were done in many regions of Saudi Arabia using various statistical methods (Al-Jaresh, 1985; Subyani, 2004; Subyani et al., 2010; Almazroui, 2011 and 2020; Almazroui & Saeed, 2020; Shwehdi, 2005; Alamodi et al., 2008; Al-Ahmad & Al-Ahmad, 2013; Tawarneh, 2015). In addition, many authors analyzed the rainfall trends as the climate change indicators over Saudi Arabia (Almazroui et al., 2012a; 2012b). However, scientific knowledge and literature relating to the climate and climate change of the SA is scattered, incomplete and limited (Hasanean & Almazroui, 2015).

2. STUDY AREA

Ha'il administrative area covers 12000 km^2, between the North latitudes

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25º30’ and 29º and between the Eastern Longitudes of 39º30’ an 44º15’ (Fig. 1). Ha’il area, located in the middle part of Northern Saudi Arabia, is classified under a hot desert climate (BWh) of the Köppen-Geiger climate classification.

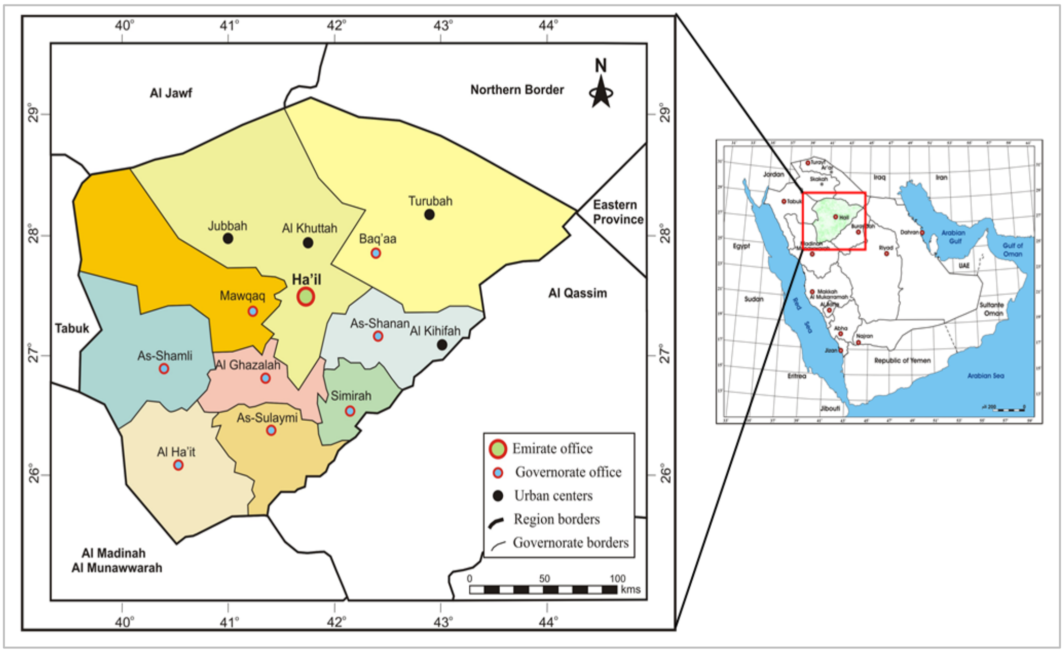

Figure 1: Geographic location of Ha’il area.

Temperatures typically range between **10.3** °C (January) and **32.7** °C (July) through the year, but rarely can drop to 0 °C or rise to as high as 44 °C. The average annual precipitation amounts to about 62 mm with 4 rainy days at Jubbah and 12 rainy days at Ha’il annually. Ha’il enjoys an average of 4440 hours of sunshine throughout the year, and daylight varies from 10 hours 30 minutes to 13 hours 48 minutes per day.

In November, it is usually that the most precipitation occurs with an average of 2 rainy days and 14.7 mm and at Ha’il, 20.2 mm at Baq’aa, 13.5 mm and 1 rainy day and 13.6 mm at Faydat Ibn Suwaylim , 12.1 mm at Al Uqlat. But, Al Ha’it and Al Ghazalah usually has the most precipitation in April with an average of 1 and 2 rainy days respectively and 17.9 mm and 11.5 mm. January is the only rainy month in Jubbah with an average of 1 rainy day and 5.0 mm. The driest months are July and August in Ha’il, Jubbah and Al Ghazalah; June and August in Al Ha’it, Faydat Ibn Suwaylim and Al Uqlat; July in Baq’aa. However, June to September is the driest period in Simirah. The sunniest months in Ha’il are May, June, July and August when the sun shines for an average of 13 hours 30 minutes a day. Least sunny months in Ha’il are November, December and January with an average of 10 hours 43 minutes of sunshine daily.

Consequently, the analysis of the seasonal rainfall detects that Spring is the season has the highest rainfall incidences, with an average of 26.1 mm, 21.1 mm, 39.7 mm, 30.0 mm, 28.2 mm, 21.9 mm at Faydat Ibn Suwaylim, Al Ghazalah, Al Ha’it, Simirah, Baq’aa and Ha’il rain stations respectively. In Al Uqlat and Jubbah, the winter is the rainy season with an average of 23.4 mm and 12.6 mm respectively. Through the summer, small quantities of precipitation are observed, while autumn received more precipitation than summer season considering the total annual rainfall. Due to its geographical location in the Central region of Northern Saudi Arabia, Ha’il area is by the El Niño Southern Oscillation (ENSO) and other circulations such as centers of high and low pressure, the North Atlantic Oscillation (NAO) and (SOI). The strength and oscillation of the subtropical jet stream play a big role in pulling hot, dry air masses of SA (Hasanean & Almazroui, 2015).

3. Materials and Methods

3.1 Description of the Dataset

The main source of rainfall data for the present study is collected from 8 rain stations selected in the Ha’il area (Tab. 1 Fig. 2). The rainfall dataset provides maximum daily rainfall and the total yearly rainfall for the entire period of 1976-2020.

The daily and annual rainfall measurements for these stations are considered the best time series which are regular in space and time in all the rain stations. Moreover, they are now supervised by the Ministry of Water, Environment and Agriculture (Tab. 2 & 3).

3.2 Rainfall Estimation for Return Periods

Probability distributions are the basic concepts in statistics. The results of rainfall statistics analysis and their probabilities of occurrence are linked by probability distributions. Rainfall data from Ha’il area were evaluated with four probability models to find the best-fit model. The probability models used are the normal (N), Log-normal (LN), Gumbel (EVI) and Exponential probability models.

In the probability distributions, the maximum value of expected rainfall (X_T) corresponding to any return period (T) can be calculated using the Chow model (Eq. 1):

$$\mathbf{X_T = X' + S_d K_T} \quad \text{(Chow et al., 1988)} \qquad [1]$$

where (X’) and (S_d) are the mean and standard deviation of (X), respectively. (K_T) is the frequency factor. It can be calculated for every probability distribution as shown in the Table 4.

Table 1: Geographic Coordinates of Studied Rain Stations.

Data collection period	Data collection years	Station No.	Code	Height (m)	Longitude (E)	Latitude (N)	Rain station
45	1976-2020	191	101 H	990	41o34'	19o51'	Ha’il
45	1976-2020	193	103 H	755	42o23'	27o52'	Baq’aa
45	1976-2020	196	106 H	920	40o56'	28o01'	Jubbah
45	1976-2020	198	108 H	910	40o25'	27o04'	Faydat ibn Suwaylim
45	1976-2020	793	105 H	950	42o07'	26o29'	Simirah
45	1976-2020	797	111 H	1064	40o27'	25o59'	Al Ha’it
45	1976-2020	508	208 H	1215	41o17'	27o06'	Al Uqlat
45	1976-2020	812	215 H	980	41o21'	26o47'	Al Ghazalah

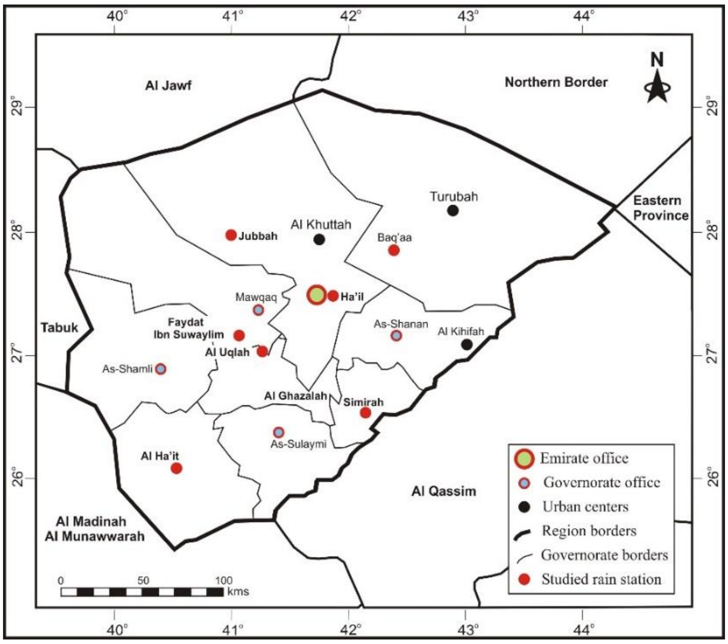

Figure 2: Geographic location of rain studied stations

Table 2: Annual Rainfall Distribution in the Studied Rain Stations.

Year	Simirah	Al Ha'it	Faydat Ibn Suwaylim	Al Ghazalah	Ha'il	Al Uqlat	Baq'aa	Jubbah
1976	146.1	9.4	57.1	90.6	256.1	204.4	4.8	39.5
1977	32.9	17.8	51.6	53.4	101.9	40	133.5	57.1
1978	2	20	28.6	31	42.4	22.2	4.3	9
1979	131.5	25.8	15.5	102.2	63.6	89.6	2.2	35.6
1980	38.6	31.5	71	67.6	31.1	75	92.8	19
1981	82	53.4	98.5	90.2	41.5	90.2	130.6	20.5
1982	212.6	134.8	100	76	35	120.2	157.9	68.8
1983	21.5	43.5	49.5	57.8	26.6	62.2	37.4	14.9
1984	2	58.9	189	142.8	65.7	202.8	232.6	36.8
1985	101	91.7	158.2	105.8	48.7	122.6	97.7	27.3
1986	85.5	105.2	132.5	141.4	65	177.2	86.4	36.4
1987	81.6	31.3	14.5	50.2	23.1	36.2	38	12.9
1988	92.5	105.2	37	45	20.7	43	45	11.6
1989	103.6	70.5	108.5	100.6	46.3	93.8	62	25.9
1990	23.7	28.4	25	34.6	15.9	52.4	20.8	8.9
1991	119.2	79.9	92	126.6	76.9	134.1	98.5	43.1
1992	107.5	35.4	75.5	22	15.7	26.4	54.8	8.8
1993	282.8	69.4	113.5	139.7	64.3	27.6	216.2	36
1994	179.5	30.5	93	78.2	36	31.2	95.5	20.1
1995	143.7	38.8	86	31.5	14.5	16.6	61	8.1
1996	71	18	113.6	21	9.7	59.8	64.1	5.4
1997	158	41.7	90	51.5	23.7	9.8	127.4	13.3
1998	68.2	26.7	95	46.6	21.4	34.6	76	12
1999	120	9.1	40	14.2	6.5	40.2	41.5	3.7
2000	64	66	44	85.5	23	18.2	3.5	12.9
2001	99	62.8	32	57.5	9	56.4	55.5	5
2002	14.2	13.8	49.6	40.5	44	30	105	4
2003	7	8.4	39.6	23.5	25	23	30	14
2004	30.5	16.5	22	47.5	24	61	9.5	30
2005	10	19.5	15	15	65	45	154.5	42
2006	37	110.5	83	11.5	76	74	125	85
2007	10.6	10	12.9	8	12	10	20.5	23
2008	74.5	68.5	82	15	65.6	64	116.5	87
2009	90	16.5	84	32	65.5	102	127.5	94
2010	52	53	56.5	66	13	74	63	27
2011	19.5	15	30.5	7	10	6	20.5	10
2012	6	27.5	45	42.4	35	41.5	7.9	19.6
2013	33	20	62	76.2	63	74.7	34.6	35.3
2014	17	37	60	37.5	31	36.8	11	17.4
2015	28	22	52	41.1	34	40.3	22	19
2016	13	16	24.5	19.4	16	19	7.9	9
2017	8	4.3	138.1	96.8	72.8	107	18.5	45.9
2018	96.5	52.1	658.6	461.9	347.3	510.5	119	53
2019	52	15	364.1	18.9	192	282.2	43	68
2020	40	15.5	45.1	11	23.8	35	8	26

Table 3: Maximum Daily Rainfall Distribution in the Studied Rain Stations.

Year	Simirah	Al Ha'it	Faydat Ibn Suwaylim	Al Ghazalah	Ha'il	Al Uqlat	Baq'aa	Jubbah
1976	29.5	9.4	12.5	30	63	33.2	4.8	10.5
1977	15.5	20.2	4.5	24.6	18.5	5	18.5	12.4
1978	2	20	14.2	8.4	14.5	15.8	4.3	4.5
1979	32.8	25.8	11	24.2	25.6	16.6	2.2	17.2
1980	12	21.5	21.5	17	6.6	22	23.2	4.5
1981	34	53.4	22	38.4	15	36.4	53.8	20.5
1982	53	37.2	24	29.4	11.5	15.8	16.8	20
1983	11.7	25	19.5	32.4	12.6	35	12.1	8.5
1984	2	30.5	81	45.6	17.8	61.6	38.7	11.9
1985	20	38.3	43.9	42.2	16.5	48.8	35.8	11
1986	17	28.2	21.5	56	21.8	52.4	31.5	14.6
1987	30	31.3	7.5	21.6	8.4	12.8	19	5.6
1988	18.5	28	17	9.6	3.7	41	18.2	2.5
1989	16	28.2	28	35.4	13.8	49.8	19	9.3
1990	9.5	28.4	15	17.4	6.8	25.2	8.5	4.5
1991	17.5	41.6	26	32.2	28.4	21.9	20	19
1992	27	30.4	12	10.8	10.2	7.8	15.3	6.8
1993	35	40.1	27	38	14.8	12.2	28	9.9
1994	25	30	21.5	18	7	18.2	16	4.7
1995	37.5	20.4	29	12	4.7	14.8	22	3.1
1996	12	15	25.5	8	3.1	11.4	30.5	2.1
1997	25	18.2	21	12	4.7	5.2	26	3.1
1998	20	12	25	15.4	6	23	35	4
1999	35	7.2	13	8.5	3.3	19	3.5	2.2
2000	26	45.3	14	47	6	17.4	10.5	4
2001	35	42.6	12	22	5	21.8	24	3.4
2002	5	10.5	25	15	18	30	40	4
2003	3	8.4	13	8.5	11	8.4	6	7.4
2004	7	9.1	11	12	9	25	3.5	15
2005	5	19.5	12	15	15	25	49	25
2006	10	42	51	6.5	15	30	21	20
2007	7	10	9	8	6	10	7.5	15
2008	26	33	38	14	34	30	22	19
2009	34	11.5	28	12.5	15	31	22	25
2010	17	19	13.5	13	4	28	22	10
2011	8	15	13	7	4	4	8	5
2012	4	12	32	7.2	5	7.1	6.3	19.6
2013	11	18	22	31.7	22	31.4	27.5	35.3
2014	3.5	28	26	10.1	7	10	8.8	17.4
2015	7	22	18	20.2	14	20	17.5	19
2016	5	16	6.4	7.2	5	7.1	6.3	9
2017	4	9	34.8	34.8	22	38.7	9.5	19.1
2018	32	30	1.2	1.1	0.8	1.2	22	11
2019	11	7	38	8	24	42.2	11	12
2020	13	12	11.1	11	7	12.3	8	10

Table 4: Frequency factor of The Probability Distributions.

Model probability distribution	K_T formula	Statistical parameters	Reference
Normal	$Z = W - \frac{2.515517 + 0.802853\,W + 0.0110328\,W^2}{1 + 1.432788\,W + 0.189269\,W^2 + 0.001308\,W^3}$ $W = \left[\ln\left\{\frac{1}{p^2}\right\}\right]^{0.5} \qquad 0 < p < 0.5$	p : Coefficient of variation.	Roberson et al., 1998
Log Normal	$K_T = \left(\frac{1}{C_v}\right)[\exp\{Z(\log(1 + C_v^2)^{0.5}) - 0.5(\log(1 + C_v^2)^{0.5})\}]$	C_v : Coefficient of variation.	Chow, 1955
Gumbel (EV1)	$K_T = -\frac{\sqrt{6}}{\pi}\left[0.5772 + \ln\left(\ln\frac{T}{T-1}\right)\right]$	T : return period.	Patel, 2020
Exponential	$Z_{eT_r} = \frac{\sqrt{6}}{\pi}(\ln Tr - 0.5772)$	T_r : return period.	Guo, 2006

As, the Chow test is a test of whether the true coefficients in two linear regressions on different data sets are equal. This test considers that linear regression for a data set (the study of the interrelationship between the dependent and independent variable of a data) is better represented if it is analyzed on two subsets of data than on the data as a whole.

3.3 Goodness Oo Fit of Probability Distributions

The best model of the probability distribution is based on the application of the Chi-square test. As, the Chi-square test is invested to identify if there is a statistically considerable difference or not among the observed frequencies and the expected frequencies across one or more classes of a contingency table.

The application of this test requires dividing the rainfall amounts into 3 or more classes in order to obtain a degree of freedom equal to at least 1 or more. The degree of freedom is calculated by applying the following equation:

df = m – p – 1 (Chow et al., 1988) [2]

Where, m is the number of classes and p is the number of estimated parameters.

The relative frequency fs(xi) is calculated from the proportion between the class frequency and the total station measurements by applying the following equation:

$$fs(xi) = \frac{n_i}{\sum n_i} \quad [3]$$

Where, n_i is the class frequency and Σn_i is the total station measurements.

The cumulative probability Fs(xi) is also calculated using the value of the standard normal variable Z appropriate for the probability distribution, as follows:

(i) Normal distribution (Bhakar et al., 2006)

$$Fs(xi) = Z = \frac{X - X'}{S_x} \quad [4]$$

Where, X is the highest value of the rainfall class, X' is the mean and S_x is the standard deviation.

(ii) Log Normal distribution (Stedinger et al., 1993)

$$Fs(xi) = Z = \frac{\ln X - a}{b} \quad [5]$$

Where :

$$a = \ln X' - \frac{\ln(1 + C_{vx}{}^2)}{2} \quad [6]$$

$$b = [\ln(1 + C_{vx}{}^2)]^{0.5} \quad [7]$$

Where, X is the highest value of the rainfall class, X' the mean and C_{vx} the standard deviation.

The cumulative probability Fs(xi) can be computed using the table of Z score for standard normal distribution as follow :

Fs(xi) = 1 – Z, if Z < 0 and Fs(xi) = Z, if Z > 0.

(iii) Gumbel distribution (EV 1) (Kindson and Richards, 2005)

$$Fs(xi) = \exp[-\exp(-y)] \quad [8]$$

Where :

$$y = \frac{x - u}{\alpha} \quad [9]$$

$$\alpha = \frac{\sqrt{6}}{\pi} Sd \quad [10]$$

$$u = x' - 0.5772\,\alpha \quad [11]$$

Where x' the mean and Sd the standard deviation.

(iv) Exponential distribution (Guo, 2006)

$$Fs(xi) = 1 - \left[-\frac{x}{a}\right] \quad [12]$$

Where a is the mean and x is the highest value of the rainfall class.

3.4 Chi-squaretest

The value of Chi-square test is the total of values obtained for every rainfall class using the following equation :

$$X_c^2 = \frac{n[f_s(x_i) - p(x_i)]^2}{p(x_i)} \quad [13]$$

Where n is total of measurement, $f_s(xi)$ the relative frequency of the rainfall class and p(xi) can be computed using the following equation :

$$P(xi) = F_s(xi) - F_s(xi - 1) \quad [14]$$

3.5 Estimation of the Return Period

The return period of any rainfall amount is computed using Gauss model by applying the following equation : (Dubreuil, 1974)

$$T = \frac{1}{1 - F(P_i)} \quad [15]$$

Where, T the return period, $F(P_i)$ is the occurrence probability for the return period, which equals:

$$F(P_i) = e^{-e^{-u}} \quad [16]$$

Where, u is Gauss variate :

$$u = \frac{P_i - P'}{\sigma P_i} \quad [17]$$

Where, P_i is the rainfall amount, P' is the mean and σP_i the standard deviation.

4. Results and Discussion

4.1 Analysis of Rainfall Spatial Distribution

The analysis of rainfall spatial distribution is based on the comparison of annual rainfall class distribution shown in Table 5 and Figure 3.

Table 5: Frequency Distribution of Annual Rainfall.

Rain stations		Below 20	20-40	40-60	60-80	80-100	100-120	Above 120	Total
Simirah	n	11	9	2	4	7	5	7	45
	%	24.4	20	4.4	8.9	15.6	11.1	15.6	100
Al Ha'it	n	17	11	6	6	5	0	0	45
	%	37.8	24.4	13.3	13.3	11.1	0	0	100
Faydat Ibn Suwaylim	n	4	9	10	3	10	3	6	45
	%	8.9	20	22.2	6.7	22.2	6.7	13.3	100
Al Ghazalah	%	9	8	11	5	4	3	5	45
	%	20	17.8	24.4	11.1	8.9	6.7	11.1	100
Ha'il	n	10	15	5	11	4	0	0	45
	%	22.2	33.3	11.1	24.4	8.9	0	0	100
Uqlat Ibn Jibrin	n	6	11	8	7	3	3	7	45
	%	13.3	24.4	17.8	15.6	6.7	6.7	15.6	100
Baq'aa	n	10	8	5	5	5	3	9	45
	%	22.2	17.8	11.1	11.1	11.1	6.7	20	100
Jubbah	n	21	14	5	2	3	0	0	45
	%	46.7	31.1	11.1	4.4	6.7	0	0	100
Total	n	88	85	52	43	41	17	34	360
	%	24.4	23.6	14.4	11.9	11.4	4.7	9.4	100

From Table 5, the frequency of annual rainfall below 40 mm constitutes the highest proportions, with 44.4%, 55.6%, 60.0%, 62.2% and 77.8% of the total annual rainfall frequency in Simirah, Ha'il, Baqa'a, Al Ha'it and Jubbah, respectively. However, the two rainfall classes (20-40 mm) and (40-60 mm) constitute 42.2% of the total annual rainfall frequency in Uqlat Ibn Jibrin and Faydat Ibn Suwaylim. However, the annual rainfall frequency at Al-Ghazala station consists mainly of rain of the two classes (below 20 mm) and (40-60 mm) with a percentage of 44.4% of the total annual rainfall frequency.

In addition, the frequency distribution of maximum daily rainfall is shown in Table 6 and Figure 4. From Table 6, the frequency of maximum daily rainfall below 20 mm constitutes the highest proportions, with 57.8%, 60.0%, 64.4%, 82.2% and 91.1% of the total maximum daily rainfall frequency in Baq'a, Al Ghazalah, Simirah, Ha'il and Jubbah, respectively. However, the rainfall classes (10-20 mm) constitute 55.6%, 60.)% and 73.3% of the total frequency of maximum daily rainfall in Uqlat Ibn Jibrin, Al Ha'it and Faydat Ibn Suwaylim, respectively.

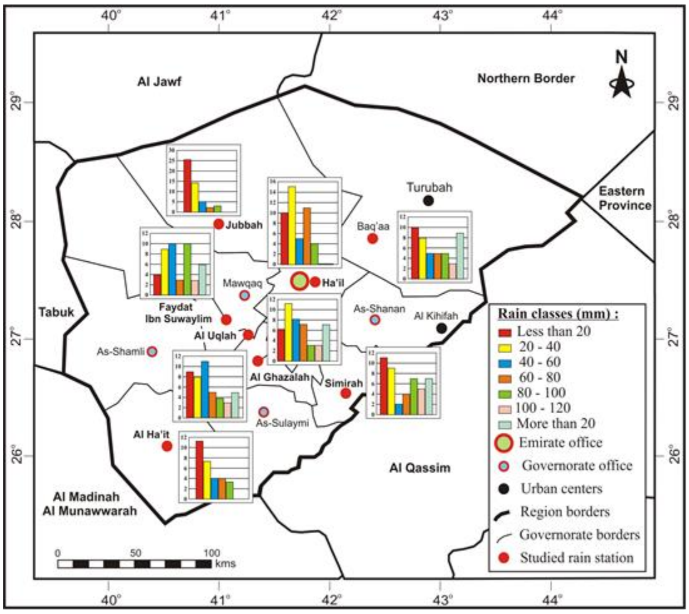

Figure 3: Spatial distribution of annual rain classes

Table 6: Frequency Distribution of Maximum Daily Rainfall.							
Rain stations		below 10	Oct-20	20-30	30-40	above 40	**Total**
Simirah	n	14	15	7	9	0	**45**
	%	31.1	33.3	15.6	20	0	**100**
Al Ha'it	n	6	14	13	6	6	**45**
	%	13.3	31.1	28.9	13.3	13.3	**100**
Faydat Ibn Suwaylim	n	5	17	16	4	3	**45**
	%	11.1	37.8	35.6	8.9	6.7	**100**
Al Ghazalah	%	12	15	7	7	4	**45**
	%	26.7	33.3	15.6	15.6	8.9	**100**
Ha'il	n	21	16	6	2	0	**45**
	%	46.7	35.6	13.3	4.4	0	**100**
Uqlat Ibn Jibrin	n	8	14	11	6	6	**45**
	%	17.8	31.1	24.4	13.3	13.3	**100**
Baq'aa	n	14	12	11	6	2	**45**
	%	31.1	26.7	24.4	13.3	4.4	**100**
Jubbah	n	21	20	4	0	0	**45**
	%	46.7	44.4	8.9	0	0	**100**
Total	n						**360**
	%						**100**

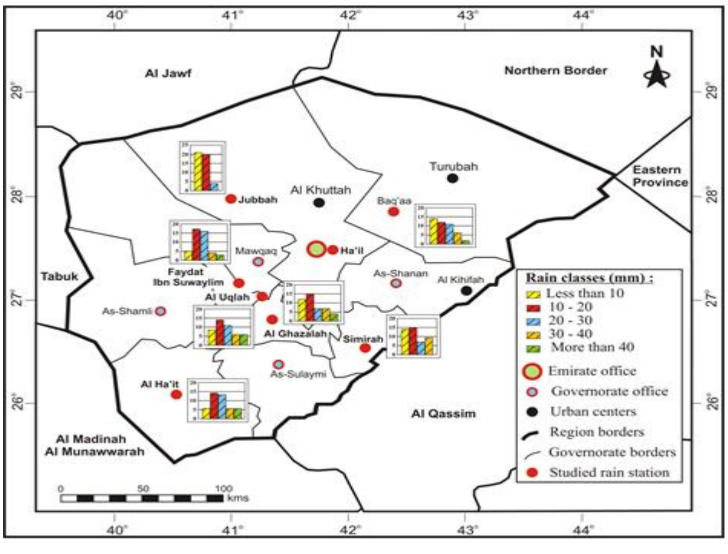

Figure 4: Spatial distribution of Daily maximum rain classes

4.2 Goodness of Fit of Probability Distributions

The Chi-square goodness of Fit test is widely used to compare the frequencies of categorical rainfall with the probability distributions using the statistical models (…), such as Normal distribution, Log Normal distribution, Gumbel (EV 1) and Exponential. This paper, applied these four probability distributions to analyze and estimate the annual and maximum daily rainfall in eight rain stations located in the Ha'il area.

In Chi-square goodness of Fit test, the term goodness of Fit is used to compare the observed rainfall distribution with the hypothesized probability distribution. So, Chi-square goodness of Fit test determines how well the theoretical distribution fits the empirical distribution. The Null hypothesis assumes no significant difference between the observed and the hypothesized probability distribution. The calculated value of Chi-square goodness of Fit test is compared with the critical value 0.05 of the significance level at the degree of freedom.

Consequently, if the calculated value of Chi-square goodness of Fit test is greater than the critical value 0.05, the null hypothesis will be rejected and there is no significant difference between the distribution of rainfall observed and the hypothesized probability distribution. The Chi-square goodness of Fit test is applied to binned rainfall data put into classes. However, the Chi-square test value depends on the binned rainfall in at least 4 classes, to obtain a degree of freedom equals 1. Therefore, the Chi-square goodness of Fit test requires a sufficient sample size in order to obtain more than 4 classes. The Chi-square is calculated for every rainfall class using the cumulative distribution function for the probability distribution being tested. The table 7 summarized the computed values of Chi-square goodness of Fit-test of annual rainfall.

Table 7: Chi Square Goodness of Fit of Annual Rainfall.

Statistics	Simirah	Al Ha'it	Faydat Ibn Suwaylim	Al Ghazalah	Ha'il	Uqlat Ibn Jibrin	Baq'a	Jubbah
X'	71.3	41	87.5	65.2	53.3	78.3	68.5	28.9
Sd	61.4	31.7	105.5	71.4	63.9	87.6	57.4	23.1
df	4	2	4	4	3	4	4	2
$X_t^2{}_{\alpha:0.05}$	9.488	5.991	9.488	9.488	7.815	9.488	9.488	5.991
ND	8.235	5.598	52.892	14.026	46.497	29.756	22.415	71.221
Log ND	14.606	4.161	24.325	3.166	123.981	**0.989***	6.893	1.387
EV 1	**7.128***	17.38	14.421	8.251	19.508	16.975	**2.261***	8.006
Exponential	8.323	**1.056***	**9.470***	**4.789***	**5.879***	5.516	2.309	**1.227***

(*) – Best probability distribution at significance level 0.05.

From the table 7, the calculated Chi-square values for the ND, EV1 and Exponential probability distributions in Simirah and Log ND, EV1 and Exponential probability distributions are smaller than the critical value 0.05 at degree of freedom 4 at Baqa'a station. By comparing the Chi-square value of the three distributions, we found that the smallest values are 7.128 and 2.261 for the Gamble probability distribution at Samira and Baqa'a, respectively. Consequently, the Gumbel probability distribution is considered the best-studied probability distribution, and is applied in estimating the annual rainfall for the return periods.

In the same context, the calculated Chi-square values for the best probability distributions at the significance level of 0.05 are the Exponential probability distribution at Al Ha'it with 1.056 and a degree of freedom 2, Faydat Bani Suwaylim with 9.470 and a degree of freedom 4, Al-Ghazalah with 4.789 and a degree of freedom 4, Ha'il with 5.879 and a degree of freedom three and Jubbah with 1.227 and a degree of freedom 2. The Log ND probability distribution is the best at Uqlat Bin Jibrin with of 0.989 and a degree of freedom 4.

The goodness of Fit test of maximum daily rainfall uses the same methodology steps. Hence, Table 8 shows the calculated Chi-square values for the

four probability distributions of maximum daily precipitation, smaller than the critical value 0.05 at Ha'il at a degree of freedom 1, Baq'a and Jubbah, at a degree of freedom 2.

Table 8: Chi Square Goodness of Fit of Maximum Daily Rainfall.

Statistics	**Simirah**	**Al Ha'it**	**Faydat Ibn Suwaylim**	**Al Ghazalah**	**Ha'il**	**Uqlat Ibn Jibrin**	**Baq'a**	**Jubbah**
X'	18	23.6	21.6	20	13	23	19	11.5
Sd	12.3	11.8	13.8	13.2	10.8	14.3	12.3	7.6
df	1	2	2	2	1	2	2	2
$X_t^2{}_{\alpha:0.05}$	3.841	5.991	5.991	5.991	3.841	5.991	5.991	5.991
ND	3.153	**0.994***	**0.989***	**0.997***	0.998	**0.996***	**0.998***	**0.998***
Log ND	2.983	3.737	6.998	3.111	1.111	1.725	3.913	6.275
EV 1	**2.571***	4.406	4.369	3.884	**0.481***	5.08	2.642	3.118
Exponential	4.267	15.586	25.589	6.044	3.7	10.569	5.921	7.041

(*) – Best probability distribution at significance level 0.05.

The values of Chi-square test of ND, Log ND and EV1 are smaller than the critical value at a degree of freedom 2 at Al Ha'it, Al Ghazalah and Uqlat bin Jibrin. The calculated Chi-square value for the two probability distributions EV1 and Log ND at Simirah and EV 1 and ND at Faydat Bani Swaylim is also reduced. By comparing the chi-square values less than the critical value 0.05, we find that the Gumbel probability distribution is the best model with the lowest Chi-square value of 2.571 and 0.481 at Samira and Hail, respectively. In the rest of the stations, the Chi-square of the Normal probability distribution is the lowest, with values ranging between 0.994 and 0.998 at the degree of freedom 2. In conclusion, the estimation of maximum daily rainfall depends on applying the Normal probability distribution in all stations, and on the Gumbel probability distribution in Simirah and Ha'il stations.

4.3 Analysis of Estimated Annual Rainfall

In Ha'il area, the rainfall is highly varying from one station to another. Statistical frequency analyses help to extract the rainfall amounts for different return periods. Ha'il region in the Northern of Saudi Arabia is selected to estimate the frequency of the annual and daily rainfall, where there are eight rainfall stations with more than 40-year recorded rainfall magnitudes. Four probability distribution functions are tested through Chi-square test of goodness-of-fit (GOF) for selecting the best statistical distribution. The findings are helpful for the estimation of rainfall for 5 to 100 years return periods. For estimating the rainfall, Chow (1954) proposed the "General equation for hydrologic frequency analysis", which applies to many probability distributions in hydrology and climatology, as follows:

$$X = X' + S_x K_T$$

X is the magnitude of a rainfall at a particular probability level, X' is the mean of the time series being used in the analysis, S_x equals the standard deviation of the time series, and K_T is the frequency factor. However, the tables 9 and Figure 5 summarized the estimated annual rainfall.

From the table 9, the predicted annual rainfall at Simirah station is ranged between 115.5 mm for the 5-year return period and 263.8 mm for the 100-year return period. The actual annual rainfall for 36 years (80%) varies between 2.0 and 107.5 mm/year, with the return periods not exceeding five years. However, only 9 annual rainfall (20%), are ranging between 119.2 mm (1991) and 282.8 mm (1993) the exceed the 5-year return period. The actual annual rainfall recorded at Al Ha'it is similar to Simirah, with the actual annual rainfall rangimg between 67.1 mm for the 5-year return period and 142.9 mm for the 100-year return period. The recorded annual rainfall for 36 years varies between 4.3 and 134.8 mm/year, with the return periods not exceeding 5 years. However, only 9 annual rainfalls, ranging between 68.5 mm (2008) and 134.8 mm (1982) exceed the 5-year return period.

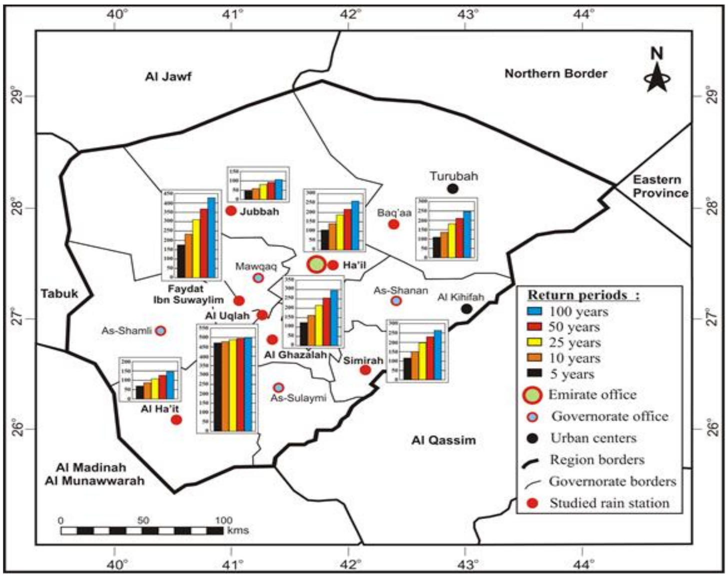

Figure 5: Spatial distribution of predicted annual rainfall in Ha'il area.

The predicted annual rainfall for the studied return periods is ranged between 105.9 mm and 263.8 mm in Ha'il and between 174.4 mm and 426.5 mm in Faydat Ibn Suwaylim. The frequency distribution of annual rainfall in Hail and Faydat Ibn Suwaylim is also similar. So that, only 3 rainfall amounts (6.7%) exceed a return period of 5 years, with quantities ranging between 192.0 mm (2019) and 347.3 mm (2018) in Ha'il and between 189.0 mm (1984); and 658.6 mm (2018) at Faydat Ibn Suwaylim. While, quantities characterized 42 years (93.3%) in the both stations with a return period not exceeding 5 years.

In Al Ghazalah, the predicted annual rainfall for the return periods varies from 124.0 mm (5-year) to 294.6 mm (100-year). But, only 5 rainfall amounts (11%) exceed 124.0 mm, with quantities ranged between 126.6 mm (1191) and 461.9 mm (2018). While, quantities characterized 40 years (89%) with a return period not exceeding 5 years. The predicted rainfall for return periods varies from 473.8 mm (5-year) to 501.2 mm (100-year) in Al Uqlat station. But 44 mounts of actual rainfall (97.8%) are less than 473.8 mm. So, only the annual rainfall recorded with 510.5 mm during 2018 is characterized by the return period exceeding 100 years. In the other hand, the lowest annual rainfall is recorded at Jubbah. The actual rainfall varies from 3.7 to 94.0 mm. However, the predicted annual rainfall is ranged between 47.9 mm (5-year) and 103.1 mm (100-year). The actual rainfall is less than 47.9 mm for 38 years (84.4%). However, only, 7 annual rainfall amounts (15.6%) are ranging between 53.0 and 94.0 mm, with a return period exceeding (5-year). In Baq'a, the actual annual rainfall varies from 2.2 and 232.6 mm. The predicted annual rainfall for the return periods increases from 109.8 mm (5-year) to 248.5 mm (100-year). So, the actual rainfall of 11 years (24.4%) exceeds 109.8 mm, with quantities ranged between 116.5 mm (2008) and 232.6 mm (1984). Consequently, 34 years are characterized by the actual rainfall values less than 116.5 mm.

4.4 Analysis of Estimated Maximum Daily Rainfall

The studied rain stations are divided, depending on the maximum daily rainfall amounts, which are characterized by return periods exceeding 5 years, into 2 groups:

(i) The first group, contains the stations of Al Ha'it, Ha'il, Al-Uqla, and Baqa'a. The actual maximum daily rainfall amounts are ranged in these stations from 7.0 to 53.4 mm, 0.8 to 63.0 mm, 1.2 to 61.6 mm and from 2.2 to 53.8 mm, respectively. The table 10 and figure 6 summarized the predicted maximum daily rainfall for the different return periods.

Accordingly, the frequency of the actual maximum daily rainfall exceeding 5-year return periods is 8 quantities (17.8%), ranging from 34.8 to 81.0 mm in Al Ha'it, 21.8 to 63.0 mm in Ha'il, 36.4 to 61.6 mm in Faydat Ibn Suwaylim and from 30.5 to 53.8 mm in Baqa'a.

(ii) The second group, contains two stations of Al Ghazalah and Jubbah. The actual maximum daily rainfall amounts are ranged in these stations from 1.1 to 56.0 mm and from 2.1 to 35.3 mm, respectively. So, the frequency of the actual maximum daily rainfall exceeding 5-year return periods is 11 quantities (24.4%), ranging from 31.7 to 56.0 mm in Al Ghazalah, and from 19.0 to 35.3 mm in Jubbah.

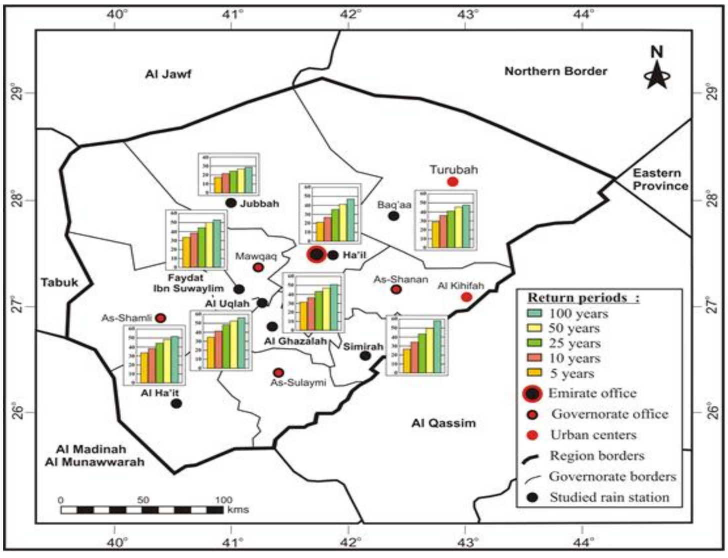

Figure 6: Spatial distribution of predicted maximum daily rainfall in Ha'il area

Table 10: The Estimated Maximum Daily Rainfall for The Return Periods.

Station	X'	Sd	Best Model	Return period				
				5-year	10-year	25-year	50-year	100-year
Simirah	18	12.3	Gumbel (EV1)	26.8	34	43.1	49.9	56.6
Al Ha'it	23.6	11.8	Normal	33.5	38.7	44.3	47.8	51.1
Faydat Ibn Suwaylim	21.6	13.8	Normal	33.2	39.3	45.8	49.9	53.7
Al Ghazalah	20	13.2	Normal	31.1	36.9	43.1	47.1	50.7
Ha'il	13	10.8	Gumbel (EV1)	20.8	27.1	35.1	41	46.9
Al Uqlat	23	14.3	Normal	35	41.3	48	52.4	56.3
Baq'aa	19	12.3	Normal	29.3	34.8	40.5	44.3	47.6
Jubbah	11.5	7.6	Normal	17.9	21.2	24.8	27.1	29.2

In addition, the maximum daily rainfall ranges from 2.0 to 53.0 mm in Simirah. But the actual maximum daily rainfall exceeding the return period (5-year) are 12 amounts (26.7%), ranged between 27.0 (1992) mm and 53.0 mm (1982). So, the actual annual rainfall during 33 years (73.3%) varies between 2.0 and 26.0 mm/year, with the return periods not exceeding 5 years. However, only nine annual rainfall (20%), are ranging between 119.2 mm (1991) and 282.8 mm (1993) the exceed the 5-year return period. In Faydat Ibn Suwaylim, the actual maximum daily rainfall amounts vary from 1.2 to 81.0 mm. But the substantial maximum daily rainfall exceeding the return period 5-years are 6 amounts (13.3%), ranged between 34.8 mm (2017) mm and 81.0 mm (1984). So, the actual annual rainfall during 39 years (86.7%) varies between 1.2 and 32.0 mm/year, with the return periods most 5 years.

4.5 Estimation of the Return Periods

The estimated return periods of actual annual rainfall are computed using Gauss variate. The frequencies and return periods of actual annual rainfall amounts are summarized and classified in the table 11 and graphically presented in figure 7.

From the table 11, the annual rainfall varies in quantities and relative distribution from one station to another. Accordingly, the rainfall that falls annually are ranging between 26.0 and 75.5 mm/year, constitutes 57.8% l in Faydat Ibn Suwaylim, Jubbah and Baqa'a stations. Those ranging between 57.8 and 64.0 mm/year, include 62.2% of the annual rainfall in Al-Ghazalah and Al Uqlat stations. While the annual rainfall doesn't exceed 64.0 mm/year, 46.3 mm/year and 37.0 mm/year constitute, 51.1% in Simirah, 64.4% in Hail, and 60.0% in Al Ha'it stations.

However, the annual rainfall for 2-year return period, varies also from one station to another. It ranges between 43.1 mm/year in Jubbah and 158.2 mm/year in Faydat Ibn Suwaylim stations. The frequency of this annual rainfall does not exceed 7 in Al Ha'it. While, it reaches a maximum of 16 times in Faydat Ibn Suwaylim. In general, the proportion of these precipitation is ranged between 15.6% in Al Ha'it and 35.6% in Faydat Ibn Suwaylim.

In addition, the annual rainfall for 3-year return period varies from 101.9 mm (1977) at Ha'il to 189.0 mm (1984) at Faydat Ibn Suwaylim stations. But, the annual rainfall frequency reaches to 6 times at Baq'a station. So, the proportion of these annual rainfall is ranged from 2.2 % at Ha'il and Baq'a to 13.3% at Faydat Ibn Suwaylim.

The rainfall for return exceeding 4 years is 6 amounts, ranging between 143.7 and 282.8 mm/year in Simirah, 79.9 and 134.8 mm/year in Al Ha'it and 57.1 and 94.0 mm/year in Jubbah stations. While, the frequency of the annual rainfall is 5 amounts, ranging between 177.2 and 510.5 mm/year in Al Uqlat, 133.5 and 232.6 mm/year in Baq'a stations. Hwever, the annual rainfall quantities do not exceed 3 times, with quantities ranging between 192.0 and 347.3 mm/year in Ha'il, twice in Faydat Ibn Suwaylim, with quantities ranging between 364.1 and 658.6 mm/year, and once, equivalent to 461.9 mm/year in Al-Ghazalah station.

Table 11: Frequency and Return Periods of Actual Annual Rainfall.

Station	Variable	Return period			
		$T_{1\text{-year}}$	$T_{2\text{-year}}$	$T_{3\text{-year}}$	$T_{\geq 4\text{-year}}$
Simirah	P(mm)	2.0 < P < 64.0	68.2 < P < 107.5	119.2 < P < 131.5	P > 131.5
	Fq	23	13	3	6
	%	51.1	28.9	6.7	13.3
	"u"	-1.1 < u < -0.1	-0.1 < u < 0.6	0.8 < u < 1.0	u ≥ 1.0
Al Ha'it	P(mm)	4.3 < P < 37.0	38.8 < P < 58.9	62.8 < P < 70.5	P > 70.5
	Fq	27	7	5	6
	%	60	15.6	11.1	13.3
	"u"	-1.2 < u < -0.1	-0.1 < u < 0.6	0.7 < u < 0.9	u ≥ 0.9
Faydat Ibn Suwaylim	P(mm)	12.9 < P < 75.5	82.0 < P < 158.2	P = 189.0	P > 189.0
	Fq	26	16	1	2
	%	57.8	35.6	2.2	4.4
	"u"	-0.7 < u < -0.1	0.0 < u < 0.7	u = 1.0	u ≥ 1.0
Al Ghazalah	P(mm)	7.0 < P < 57.8	66.0 < P < 105.8	126.6 < P < 142.8	P > 142.8
	Fq	28	12	4	1
	%	62.2	26.7	8.9	2.2
	"u"	-0.8 < u < -0.1	0.0 < u < 0.6	0.9 < u < 1.1	u ≥ 1.1
Ha'il	P(mm)	4.3 < P < 37.0	38.8 < P < 58.9	62.8 < P < 70.5	P > 70.5
	Fq	29	12	1	3
	%	64.4	26.7	2.2	6.7
	"u"	-0.7 < u < -0.1	-0.1 < u < 0.4	u = 0.8	u ≥ 0.8
Al Uqlat	P(mm)	6.0 < P < 64.0	74.0 < P < 134.1	P = 0	P > 134.1
	Fq	28	12	0	5
	%	62.2	26.7	0	11.1
	"u"	-0.8 < u < -0.2	0.0 < u < 0.6	0	u ≥ 0.6
Baq'a	P(mm)	2.2 < P < 63.0	64.1 < P < 105.0	116.5 < P < 130.6	P > 130.6
	Fq	26	8	6	5
	%	57.8	17.8	13.3	11.1
	"u"	-1.2 < u < -0.1	-0.1 < u < 0.6	0.8 < u < 1.1	u ≥ 1.1
Jubbah	P(mm)	3.7 < P < 26.0	27.0 < P < 43.1	45.9 < P < 53.0	P > 53.0
	Fq	26	11	2	6
	%	57.8	24.4	4.4	13.3
	"u"	-1.1 < u < -0.1	-0.1 < u < 0.6	0.7 < u < 1.0	u ≥ 1.0

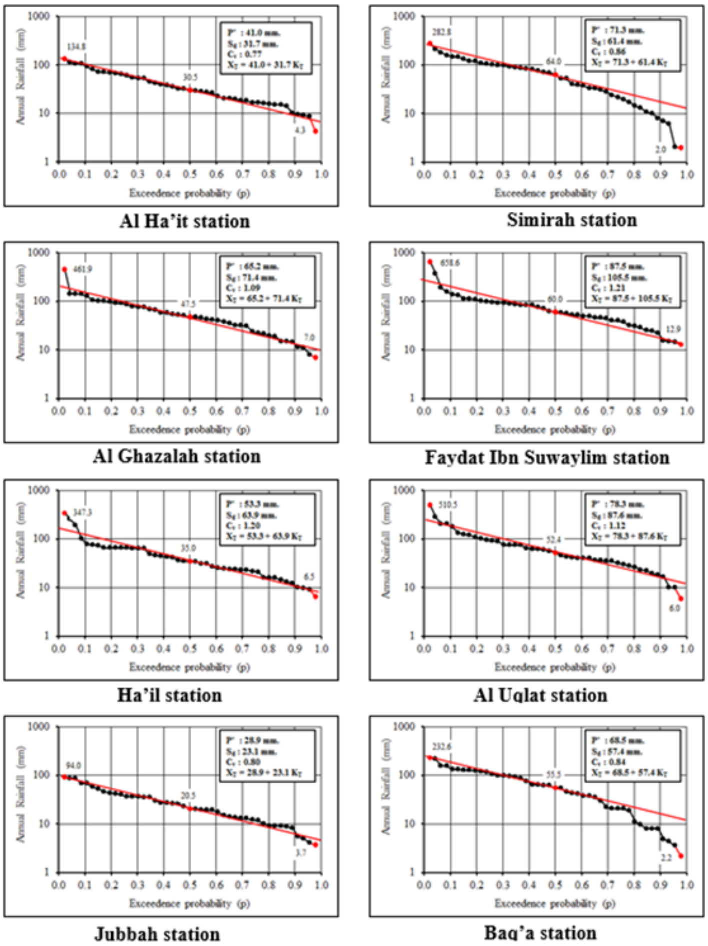

Figure 7: Exceedence probability of annual rainfall at the studied stations.

Table 12: Frequency and Return Periods of Actual Maximum Daily Rainfall.

Station	Variable	Return period			
		$T_{1\text{-year}}$	$T_{2\text{-year}}$	$T_{3\text{-year}}$	$T_{\geq 4\text{-year}}$
Simirah	P(mm)	2.0 < P < 16.0	17.0 < P < 26.0	27.0 < P < 30.0	P > 30.0
	Fq	23	10	3	9
	%	51.1	22.2	6.7	20
	"u"	-1.3 < u < -0.2	-0.1 < u < 0.7	0.7 < u < 1.0	u ≥ 1.0
Al Ha'it	P(mm)	7.0 < P < 22.0	25.0 < P < 31.3	P = 33.0	P > 33.0
	Fq	24	12	1	8
	%	53.3	26.7	2.2	17.8
	"u"	-1.4 < u < -0.1	-0.1 < u < 0.7	u = 0.8	u ≥ 0.8
Faydat Ibn Suwaylim	P(mm)	1.2 < P < 19.5	21.0 < P < 29.0	32.0 < P < 34.8	P > 34.8
	Fq	22	16	2	5
	%	48.9	35.6	4.4	11.1
	"u"	-1.5 < u < -0.2	0.0 < u < 0.5	0.8 < u < 1.0	u ≥ 1.0
Al Ghazalah	P(mm)	1.1 < P < 18.0	20.2 < P < 24.6	29.4 < P < 32.4	P > 32.4
	Fq	27	5	5	8
	%	60	11.1	11.1	17.8
	"u"	-1.4 < u < -0.2	0.0 < u < 0.3	0.7 < u < 0.9	u ≥ 0.9
Ha'il	P(mm)	0.8 < P < 11.5	12.6 < P < 18.5	21.8 < P < 24.0	P > 24.0
	Fq	24	13	4	4
	%	53.3	28.9	8.9	8.9
	"u"	-1.01 < u < -0.1	0.0 < u < 0.5	0.8 < u < 1.0	u ≥ 1.0
Al Uqlat	P(mm)	1.2 < P < 20.0	21.8 < P < 31.4	33.2 < P < 36.5	P > 36.4
	Fq	22	13	3	7
	%	48.9	28.9	6.7	15.6
	"u"	-1.5 < u < -0.2	-0.1 < u < 0.6	0.7 < u < 0.9	u ≥ 0.9
Baq'a	P(mm)	2.2 < P < 17.5	18.2 < P < 26.0	27.5 < P < 31.5	P > 31.5
	Fq	21	14	4	6
	%	46.7	31.1	8.9	13.3
	"u"	-1.4 < u < -0.1	-0.1 < u < 0.6	0.7 < u < 1.0	u ≥ 1.0
Jubbah	P(mm)	2.1 < P < 10.5	11.0 < P < 15.0	17.2 < P < 19.6	P > 19.6
	Fq	24	8	7	6
	%	53.3	17.8	15.6	13.3
	"u"	-1.2 < u < -0.1	-0.1 < u < 0.5	0.8 < u < 1.1	u ≥ 1.1

The return periods for the actual maximum daily rainfall were estimated by the same method (Table 12 and Figure 8). The table 12 summarizes the frequency distribution and the return periods in every station.

From this table, the frequency of recorded rains every year varies from one station to another. The frequency of these rains ranges between 21 and 27 times, equivalent respectively to 46.7% and 60.0% of the maximum daily rainfall amounts observed during the study period (45 years) in the Baqa'a and Al-Ghazalah stations. The frequency for the return period 2 years also ranges between 5 and 16 times, equivalent respectively to 11.1% and 35.6% of the maximum daily rainfall amounts, at Al-Ghazalah and Faydat Ibn Suwaylim stations.

In addition, the frequency of rain for the return period 3-year is ranged between once at and 7 times, equivalent to 2.2% and 15.6% of the maximum daily rainfall at Al Ha'it and Jubbah respectively. These rainfall amounts vary from 19.6 mm/day at Jubbah to 36.4 mm/day at Al Uqlat stations. However, maximum daily rainfall for the frequency of 4 years and more is relatively homogeneous, with more than 20 mm/day, ranging between 4 times at Ha'il and 9 times in Simirah station.

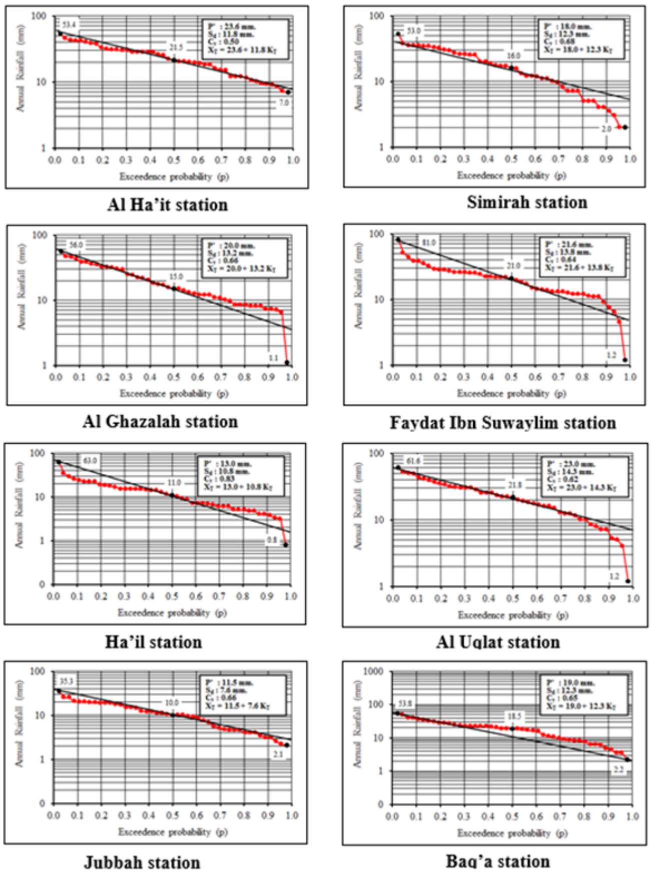

Figure 8: Exceedence probability of maximum daily rainfall at the studied

5. Conclusion

Such as arid regions Ha'il area suffers from the scarcity of rain gauges and the homogeneous record length limitation of observed rainfall data, which are usually short to ensure reasonable estimates. The frequency analysis is one of the most statistical approaches used to compensate the lack or limitation of data availability. However, The frequency analysis approaches have not yet been investigated in arid regions. Annual and maximum daily rainfall data recorded during 45 years (1976-2020) at eight daily rainfall stations in the Ha'il region, North of Saudi Arabia are processed to estimate the annual and the maximum daily rainfall using four probability distributions (Normal, Log Normal, Gumbel and Exponential). Computation of mean, standard deviation and coefficient of variation showed high variability of the region's annual and maximum daily rainfall data. The purpose of the study was to find the best probability distribution of annual and maximum daily rainfall at every rainfall gauging station using the goodness of fit of Chi-square test.

The goodness of fit of Chi-square test showed the best probability distributions of annual rainfall were Gumbel (EV1) at Simirah, and Baq'a stations; Exponential at Al Ha'it, Faydat Ibn Suwaylim, Al Ghazalah and Jubbah stations and log Normal at Al Uqlat station. The goodness fit of the same test showed the Normal distribution as the best probability distribution of maximum daily rainfall at all rain stations, except Simirah and Ha'il. The Gumbel (EV1) probability distribution is the best probability distribution in these two rain stations.

The annual rain estimations for 5-year return period vary from 47.9 mm at Jubbah to 473.8 mm at Al Uqlat; and for 100-year return period from 103.1 mm at Jubbah to 501.2 mm at Al Uqlat. However, the maximum daily rain estimations for 5-year return period vary from 17.9 mm at Jubbah to 35.0 mm at Al Uqlat; and for 100-year return period from 29.2 mm at Jubbah to 56.6 mm at Simirah.

The return periods for the actual annual rainfall calculated by the Gauss model, showed that the proportion of annual rainfall that recur every year ranged between 51.1% and 64.4% of the total observed amounts in Samira and Ha'il stations, respectively. While the proportion of the quantities of rain that recurs once a year are ranged between 15.6% and 35.6% of the total rain quantities observed respectively in the two stations of Al Ha'it and Faydat Ibn Suwaylim. Also, the proportion of the rain quantities for return period 3-years vary respectively from 2.2% and 13.3% of the total quantities recorded in Faydat Ibn Suwaylim, Ha'il and Baqa' stations. In addition, the proportion of the rain quantities for the return periods exceeding 4 years varies respectively between 2.2% and 13.3% of the observed rain in Al-Ghazalah, Simirah, Al-Ha'it and Jubbah stations.

On the other hand, the proportion of maximum daily rainfall can be occurred every year varies respectively from 46.7% and 60.0% of the total observed amounts at Baq'a and Al Ghazalah stations. At the same time, the proportion of the rain quantities that recurs twice a year are ranged between 11.1% and 35.6% of the total rain quantities observed respectively in the two stations of Faydat Ibn Suwaylim and Al Ghazalah stations. Also, the proportion of the rain quantities for the return period of 3-years varies respectively from 2.2% and 15.6% of the total amounts recorded in Al Ha'it and Jubbah stations. Finally, the proportion of the rain quantities for the return periods exceeding 4-years varies between 8.9% and 20.0% of the observed rain in Ha'il and Al Uqlat stations respectively.

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